ADVANCED CAPITAL BUDGETING

Introduction:-

B/S

Liabilities	Assets
Financing Decision →	Investing Decision
$\boxed{E+D}$ $D/E = ?$	

⇓

Capital Budgeting Decision

⇓

NPV Decision

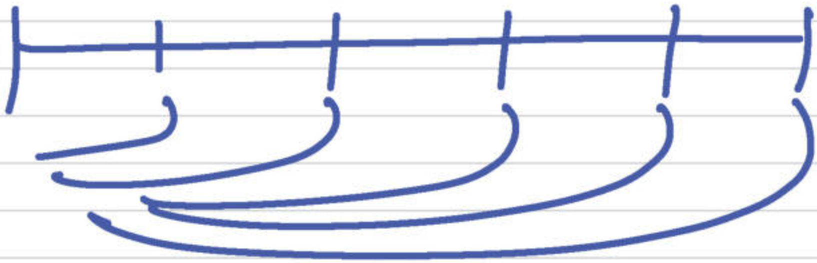
+

Risk Analysis

⇓

Risk under Capital Budgeting.

NPV ⇒



NPV = PV of CI's − PV of CO's / Initial Investment

\+

Risk Under Capital Budgeting.

1) Probability Distribution Approach:-

   S.D | Variance | CV

2) Conventional Techniques

   (i) RADR

   (ii) CEC

3) Other Techniques:-

(i) Sensitivity Analysis

(ii) Scenario Analysis

4) Inflation Under Cap. Bug.

(IFM)

5) Replacement Decision:- | Replacement Cycle:-

Evaluation between two proposals:-

6) Real options under Capital Budgeting.

(i) Risk-Neutral

(ii) BSM

7) Adjusted NPV Method:-

8) Backward Decision Tree Approach:-

Concept: Risk Analysis under Capital Budgeting:-

NPV / Return ⇒ 1,00,000

⇓

Expected NPV

(+) Risk

Variation in NPV

₹900

Techniques:- (Risk Analysis)

- Conventional Techniques
  - → RADR
  - → CEC
- Statistical Techniques (Imp)
  - → Prob. Distribution Approach:-
    - → Variance (σ^2)
    - → S.D (σ)
    - → CV
    - → S.D/Variance
      - Hiller's
      - Hertz's

      + Decision Tree Approach
- Other Techniques
  - → Sensitivity Analysis
  - → Scenario Analysis

1) Conventional Techniques :-

RADR Method

(Risk Adjusted DR) method

[Risk is adjusted into the Discount Rate]

CEC Method

(Certainity Equivalent Co-efficient Method)

[Risk is adjusted into the Cashflows]

1) RADR Method :-

This method adjusts the risk in the Discount Rate.

CF's → Cal. → Risky CF's

(Uncertain)

Discounting → DR → RADR

Risk Adjusted NPV ✓

⇒ There are several methods for Cal. of RADR depending on the information given in the question:-

1) RADR = Risk-free rate + Risk Premium

OR

$(1+RADR) = (1+RFR)(1+\text{Risk Premium})$

2) RADR = K_0 + Extra Risk Premium

DR

Note: If project is more risky than we can add extra risk premium into it.

And if the project is less risky than we should deduct extra risk premium.

3) RADR may be calculated by using CV i.e. Co-effecient of Variation of the project.

$$\frac{\text{Risk}}{\text{Return}} \Rightarrow \frac{\text{S.D}}{\text{Avg. Return / Exp. Return}}$$

$\boxed{\uparrow \text{CV} \uparrow \text{Risk}}$ $\boxed{\downarrow \text{CV} \downarrow \text{Risk}}$

4) Inf. RADR can be calculated using the CAPM: -

$$E(R) = R_F + \beta [R_m - R_F]$$

β → Risk Index

⇒ Imagine the firm to be the mkt. portfolio

K_0 can be assumed / treated as R_m.

$$RADR = R_F + \Delta [K_0 - R_F]$$

Δ → (Risk Index)

Δ = Each project sensitivity with the firm

Q.1A

I Cal. of NPV using Rf rate:-

Year	CFs	RFR = 7%	PV
1	25	0.935	23.375
2	60	0.873	52.38
3	75	0.816	61.20
4	80	0.763	61.04
5	65	0.713	46.345

Pvg CI → 244.34

Less: Initial Invt. → 100.00

NPV based on RF rate ₹ 144.34 lakhs

II Cal. of Risk Adjusted NPV:- (As per ICAI)

RADR = RFR + Risk Premium

$\Rightarrow$ 7% + 7% = 14% (Ideally)

Year	CF's	PVF@14%	PV
1	25	0.877	21.925
2	60	0.769	46.14
3	75	0.675	50.625
4	80	0.592	47.36
5	65	0.519	33.735

PV of CI's ₹ 199.79

(−) Initial Invt. ₹ 100.00

Risk Adj. NPV ₹ 99.79 Ideally

Q.1C

Project 'X'

CV = 1.2 → RADR = 16%

Cal. of Risk Adj. NPV :—

⇒ 70,000 [PVAF @ 16%, 5 years]
(3.274)

(−)

₹ 210,000

⇒ ₹ 19,180 ✓

Project 'Y'

CV = 0.80 → RADR = 14%

NPV ⇒ 42000 × PVAF @ 14%, 5 years
(3.433)

(−)

1,20,000

$\Rightarrow$ ₹ 24,186 ✓

Project 'Z' :-

CV = .40 → RADR = 12%

$\Rightarrow$ 30,000 × PVAF @ 12%, 5 years (3.605)

(−) 1,00,000

$\Rightarrow$ ₹ 8150

Q.1E

$R_F = 10\%$ $K_0 = 15\% = \underline{R_M}$

(i) **RADR** $\Rightarrow R_F + \beta\,[K_0 - R_F]$

$P\text{I} \Rightarrow \Rightarrow .10 + 1.80\,[.15 - .10]$

$\Rightarrow 19\%$

$P\text{II} \Rightarrow .10 + 1\,[.15 - .10] \Rightarrow 15\%$

$P\text{III} \Rightarrow .10 + .60\,[.15 - .10] \Rightarrow 13\%$

(ii) **Cal. of Risk Adjusted NPV:-**

NPV $\Rightarrow$ 6,00,000 × PVAF @ 19%, 4 years

(−)

15,00,000

$\Rightarrow$ 6,00,000 × 2.639 − 15,00,000

NPV ⇒ ₹83,400

PII

Year	CI's	PVF@15%	PV
1	6,00,000	·870	5,22,000
2	4,00,000	·756	302400
3	5,00,000	·658	329000
4	2,00,000	·572	114400
			12,67,800
Less: Cost of Investment			11,00,000
		NPV	1,67,800

PIII

Year	CI's	PVF@13%	PV

1	4,00,000	·885	354000
2	6,00,000	·783	469800
3	8,00,000	·693	554400
4	12,00,000	·613	735600
		PV of CI's	21,13,800
		(−) Initial Inv.	19,00,000
		NPV	2,13,800

Decision:

Project III has highest NPV. So, it should be accepted by the firm.

2) CEC Method:-

[Certainity Equivalent Co-efficient]

In this method, risk is adjusted in cash flows.

[Given Certainity Equivalent Co-efficient.]

Example

Year 0	60,000	80,000	1,00,000
-1,00,000			
CEC 1	.80	.70	.50

Certain CF's (RF CF's)

DR → RF rate

Risk Adjusted NPV

Step 1: Cal. Certain CF's using CEC factor.

Step 2 Use RF rate as DR

Step 3: Cal. Risk Adjusted NPV

CEC factor at Time = 0 should be 1 or 100%.

Q.1B

(₹ lakhs)

Year	CF's	CFC factor	Certain CF's	DR=5%	PV
0	(45)	1	(45)	1	(45)
1	10	.90	9	.952	8.568
2	15	.85	12.75	.907	11.564
3	20	.82	16.40	.864	14.17
4	25	.78	19.50	.823	16.05

Risk-Adj: NPV ₹ 5.352

Q.1D

Project M

Year	CF's	CEC	Certainty CF's	DR=6%	PV
0	(8,50,000)	1	(8,50,000)	1	(8,50,000)
1	4,50,000	.80	3,60,000	.943	339480
2	5,00,000	.70	3,50,000	.890	311500
3	5,00,000	.50	2,50,000	.840	210,000

Risk Adj. NPV (+) 10980 ✓

Project N

Year	CF's	CEC	Certain CF's	DR=6%	PV
0	(825000)	1	(825000)	1	(825000)
1	4,50,000	.90	405000	.943	381915
2	4,50,000	.80	3,60,000	.890	320400

3 5,00,000 .70 3,50,000 .840 294000

Risk Adj. NPV (+) 171315

Decision:-

Project 'N' should be selected because Risk-Adj: NPV is greater in this case.

(ii) If RADR method is used, Project 'M' should be analysed with higher rate because Certainty factor is less in this case i.e. Risk is higher & if risk is higher, Discount rate will be higher.

Concept: Probability Distribution Approach:-

OR

Statistical Techniques:-

1) Expected CF's:-

Assumptions	CF's	Prob.	Exp. CF's $\sum CF_i \times Prob$
Best	4,00,000	·40	4,00,000 × ·40
Avg.	3,00,000	·50	3,00,000 × ·50
Worst	2,00,000	·10	2,00,000 × ·10
		1	Σ = 3,30,000

2) Expected NPV:-

Alt. 1 Cal. of Exp. NPV using Exp. CF's

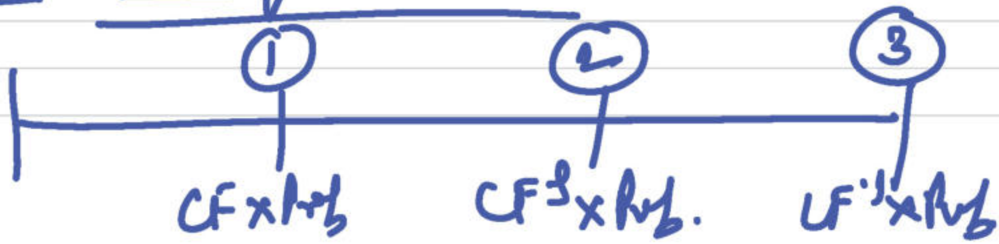
Case I: Single Period:-

CF's	Prob.
✓	✓
✓	✓
✓	✓

Exp. CF's

$$\text{Exp. NPV} = \frac{\text{Exp. CF's}}{(1+r)^n} - \text{Initial Investment}$$

Case II: Multiple Period:-



Exp CF's 1 Exp CF's 2 Exp CF's 3

PV

Exp. NPV :-

$$\frac{\text{Exp. CF}_1}{(1+DR)^1} + \frac{\text{Exp. CF}_2}{(1+DR)^2} + \frac{\text{Exp. CF}_3}{(1+DR)^3}$$

(−)

Initial Investment

Expected NPV

Q-2A

1 year

1

$$\text{Exp. CF's} = \frac{\Sigma\, CF's \times Prob}{(1+DR)^1}$$

$$\boxed{\frac{\text{Exp. CF's}}{(1+DR)^1} - \text{Initial Inv.} = \text{Exp. NPV}}$$

Project A

Possible Event	CF's	Prob.	Exp. CF's (1)
A	8000	.10	8000 × .10
B	10,000	.20	10,000 × .20
C	12000	.40	12000 × .40
D	1400	.20	14000 × .20

E	16000	.10	$16000 \times .10$
		$=$	$\Sigma = 12000$

Exp. NPV

$$\frac{12000}{(1+.10)^1}$$

$$\Rightarrow \frac{12000}{(1+.10)^1} - 10,000$$

$$\Rightarrow 12000 \times .909 - 10,000$$

$$\Rightarrow (+)\ \underline{908}$$

Project 'B'

Possible Event	CF's	Prob.	Exp. CF's
A	24000	.10	$24000 \times .10$

B	20,000	$\cdot 15$	$20{,}000 \times \cdot 15$
C	16000	$\cdot 50$	$16000 \times \cdot 50$
D	12000	$\cdot 15$	$12000 \times \cdot 15$
E	8000	$\cdot 10$	$8000 \times \cdot 10$
		1	16000 ✓

$$\frac{16000}{(1+\cdot 10)^1}$$

$$\text{Exp. NPV} = \frac{16000}{(1+\cdot 10)^1} - 10{,}000$$

$$\Rightarrow 16000 \times \cdot 909 - 10{,}000$$

$$\Rightarrow ₹\,\underline{\underline{4544}}$$ ✓

Q.28 (SM)

Year 1

CF-S	Prob.	Exp. Value
2000	.10	2000 x .10
4000	.20	4000 x .20
6000	.30	6000 x .30
8000	.40	8000 x .40
	1	$\Sigma \Rightarrow 6000$

Year 2

CF-S	Prob.	Exp. CF-S
2000	.20	2000 x .20
4000	.30	4000 x .30
6000	.40	6000 x .40
8000	.10	8000 x .10

$\Sigma = 4800$

Year 3:-

CF's	Prob.	Exp. CF's
2000	.30	$2000 \times .30$
4000	.40	$4000 \times .40$
6000	.20	$6000 \times .20$
8000	.10	$8000 \times .10$
	1	$\Sigma = 4200$

PV of Exp. CF's

$$\frac{6000}{(1+.10)^1} \qquad \frac{4800}{(1+.10)^2} \qquad \frac{4200}{(1+.10)^3}$$

PV of Exp. CF's

$\Rightarrow 6000 \times .909 + 4800 \times .826$

$+ 4200 \times .751$

$\Rightarrow$ ₹ $\underline{\underline{12573}}$

Exp. NPV:-

$\Rightarrow$ PV of Exp. CF's − Initial Invt.

$\Rightarrow 12573 - 10,000$

$\Rightarrow$ ₹ $\underline{\underline{2573}}$

⇒ Cal. of Variance & S.D ⇒ [Exp. CF's]

Variation in CF's

(i) Variance ⇒ $\sum(\underbrace{X}_{\text{Given CF's}} - \underbrace{\bar{X}}_{\text{Exp CF's}})^2 \times \text{Prob.}$

(ii) $S.D = \sqrt{\sum(X-\bar{X})^2 \text{ Prob.}}$

(iii) CV ⇒ $\frac{\text{Risk}}{\text{Return}}$ ⇒ $\boxed{\frac{S.D.}{\text{Expected CF's}}}$

Q 2C

Cal. of Exp. CF-s :-

Project A :

Possible Event	(X) CF-s	Prob	Exp. CF-s	$(X-\bar{X})^2$ Prob.
A	8000	·10	8000 × ·10	$(8000-12000)^2 \times ·1$
B	10,000	·20	10,000 × ·20	$(10,000-12000)^2 \times ·2$
C	12000	·40	12000 × ·40	$(12000-12000)^2 \times ·$
D	14000	·20	14000 × ·20	$(14000-12000)^2 \times ·$
E	16000	·10	16000 × ·10	$(16000-12000)^2 \times ·$

$\bar{X} = 12000$

$\Sigma \Rightarrow 48,00,000$

Variance = 48,00,000

S·D = $\sqrt{48,00,000}$ ⇒ 2190.90

CV = $\frac{S \cdot D}{Exp.\ CF\text{'}s}$ ⇒ $\frac{2190.90}{12000}$

⇒ 0.183

Project 'B'

CF's (Y)	Prob.	Exp. CF's	$(Y-\bar{Y})^2$ Prob.
24000	·10	24000 × ·10	$(24000-16000)^2 \cdot 10$
20,000	·15	20,000 × ·15	$(20000-16000)^2 \cdot 15$
16000	·50	16000 × ·50	$(16000-16000)^2 \cdot 50$
12000	·15	12000 × ·15	$(16000-12000)^2 \times \cdot 15$
8000	·10	8000 × ·10	$(8000-12000)^2 \times \cdot 10$

L $\Sigma = 16000$ $\Sigma = 176,00,000$

$(\bar{y})$

Variance $\Rightarrow$ 176,00,000 $= (\sigma^2)$

$\sigma \Rightarrow \sqrt{176,00,000} \Rightarrow \underline{\underline{4195.23}}$

$CV \Rightarrow \dfrac{4195.23}{16000} \Rightarrow \underline{\underline{0.2622}}$

Select Project 'A'

Alt. 2 When Estimate NPV with Prob. are given

1) Estimate NPV (X)	Prob.	Exp. NPV
✓	✓	NPV × Prob.
✓	✓	NPV × Prob.
✓	✓	NPV × Prob.
✓	✓	NPV × Prob.
		($\bar{X}$) Σ = Exp. NPV

1) Exp. NPV ⇒ Σ NPV × Prob.

2) Variance ⇒ $(X-\bar{X})^2 \times$ Prob.

given

NPV Exp. NPV

3) $S \cdot D = \sqrt{\sum (X - \bar{X})^2 \text{ Prob.}}$

4) $CV = \dfrac{S \cdot D}{\text{Exp. NPV}}$

CV ↑ S·D ↑ Variance ↑ Risk ↑

&

vice-versa

Q 2D [SM] Imp.

W.No. 1

Project 'A'

NPV Estimate (X)	Prob.	Exp. NPV	$(X-\bar{X})^2$ Prob.
15000	·20	15000 × ·20	$(15000-9000)^2 \times ·20$
12000	·30	12000 × ·30	$(12000-9000)^2 \times ·30$
6000	·30	6000 × ·30	$(6000-9000)^2 \times ·30$
3000	·20	3000 × ·20	$(3000-9000)^2 \times ·20$
	1	$\bar{X} \Rightarrow 9000$	$\Sigma \Rightarrow 198,00,000$
		Expected NPV	= Variance

Project 'B'

Estimate	Prob.	Exp. NPV	$(Y-\bar{Y})^2$ Prob.

NPV(Y)

15000	·10	15000 × ·10	$(15000-9000)^2 \times ·10$
12000	·40	12000 × ·40	$(12000-9000)^2 \times ·40$
6000	·40	6000 × ·40	$(6000-9000)^2 \times ·40$
3000	·10	3000 × ·10	$(3000-9000)^2 \times ·10$
	1	$\bar{Y} \Rightarrow 9000$	$\Sigma = 144,00,000$

Variance

(i) Exp. NPV ⇒ A = 9000 ✓
B = 9000

(ii) S.D A ⇒ $\sqrt{\Sigma(X-\bar{X})^2 \text{ Prob.}}$

$= \sqrt{198,00,000}$

⇒ 4450 ✓

$S.D_B \Rightarrow \sqrt{144,00,000}$

$\Rightarrow 3795$

(iii) $PI \Rightarrow \frac{PV\ of\ CI's}{PV\ of\ CO's}$

$\Rightarrow \frac{NPV + PV\ of\ CO's}{PV\ of\ CO's}$

$A = \frac{9000 + 36000}{36000} \Rightarrow 1.25$

$B \Rightarrow \frac{9000 + 30,000}{30,000} = 1.30$

(iv) $CV \Rightarrow \frac{S.D}{Return \rightarrow Exp.\ NPV}$

$$A \Rightarrow \frac{4450}{9000} \Rightarrow 0.494$$

$$B = \frac{3795}{9000} \Rightarrow 0.422$$

Project 'B' is preferable because CV is less in this case.

CV tells us the risk per unit of return, so lower CV means lower risk.

Hence, Project 'B' is preferable

Concept: Decision Tree Approach:-

Type 1

Moderately Correlated CF's i.e. $r < 1$

Type 2

Perfectly Correlated CF's i.e. $r = 1$

Type 3

We can't make Decision Tree i.e. In each year CF's are different & their prob. are given

⇓ Cal. of S.D.

- Hillier's Model
- Hertz's Model

The focus of discussion is the correlation between the CF's of different year:-

Type 1. Moderately Correlated CF's :-

i.e. $r < 1$ (Decision Tree Approach)

		Year 1		Year 2		
−1,00,000	.60	70,000	.50	60,000	① Estimate NPV	.30
			.30	70,000	②	.18
			.20	80,000	③	.12
			1			
	.40	90,000	.30	70,000	④	.12
			.40	80,000	⑤	.16
			.30	90,000	⑥	.12
	1		1			1

Type 2

If $r = 1$ Perfectly Correlate CF's

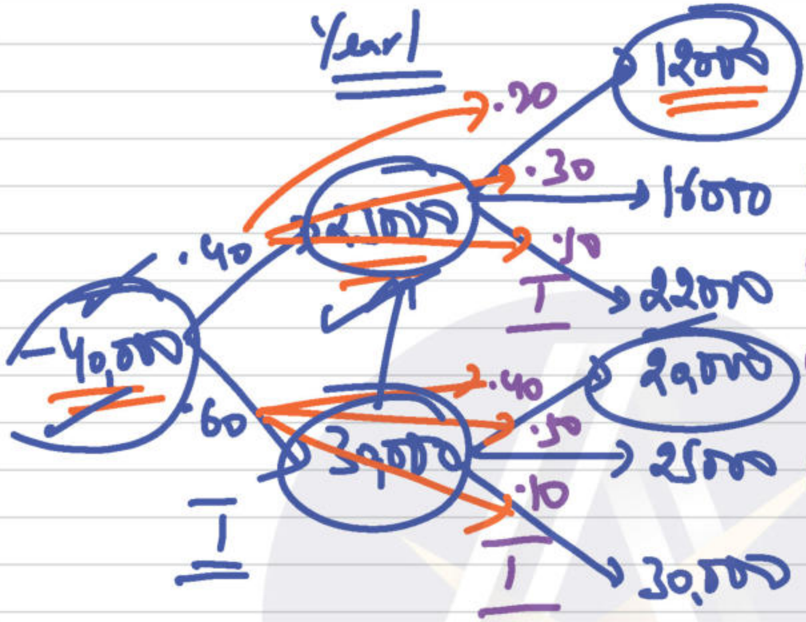
−1,00,000	.60	70,000	100%	70,000	70,000
	.40	90,000	100%	90,000	90,000

$r = 1$

Note: Type 1:- [$r<1$ means]

This means that II^{nd} year CF's depends upon the 1^{st} year CF's but the dependency is not one to one.

Q.3A Type 1:- r<1



Path	Estimate NPV	JP
1		0.08
2		0.12
3		0.20
4		0.24
5		0.30
6		0.06
		1

W.No.1 Cal. Estimate NPV:-

(1) $-40,000 + \frac{25000}{(1+.10)^1} + \frac{12000}{(1+.10)^2} \Rightarrow (-)\ 7363$

.909 .826

(2) $-40,000 + \frac{25000}{(1+.10)} + \frac{16000}{(1+.10)^2} \Rightarrow (-)\ 4059$

(3) $-40{,}000 + \frac{25000}{(1+.10)^1} + \frac{22000}{(1+.10)^2} \Rightarrow (+)\ 897$

(4) $-40{,}000 + \frac{30{,}000}{(1+.10)^1} + \frac{20{,}000}{(1+.10)^2} \Rightarrow (+)\ 3790$

(5) $-40{,}000 + \frac{30{,}000}{(1+.10)^1} + \frac{25000}{(1+.10)^2} \Rightarrow +7920$

(6) $-40{,}000 + \frac{30{,}000}{(1+.10)^1} + \frac{30{,}000}{(1+.10)^2} \Rightarrow +12050$

(b) If worst outcome is realized, the NPV which the project yield is (–) 7363.

The Probability of occurance of

the NPV is 0.08 i.e. 8%

Extra Part:

If required in question:-

Expected loss = −7363 × 8%

⇒ 589.04

(c) If Best outcome is realized, the NPV which the project yield is +12050

The probability of occurrence of the NPV is 0.06 i.e. 6%

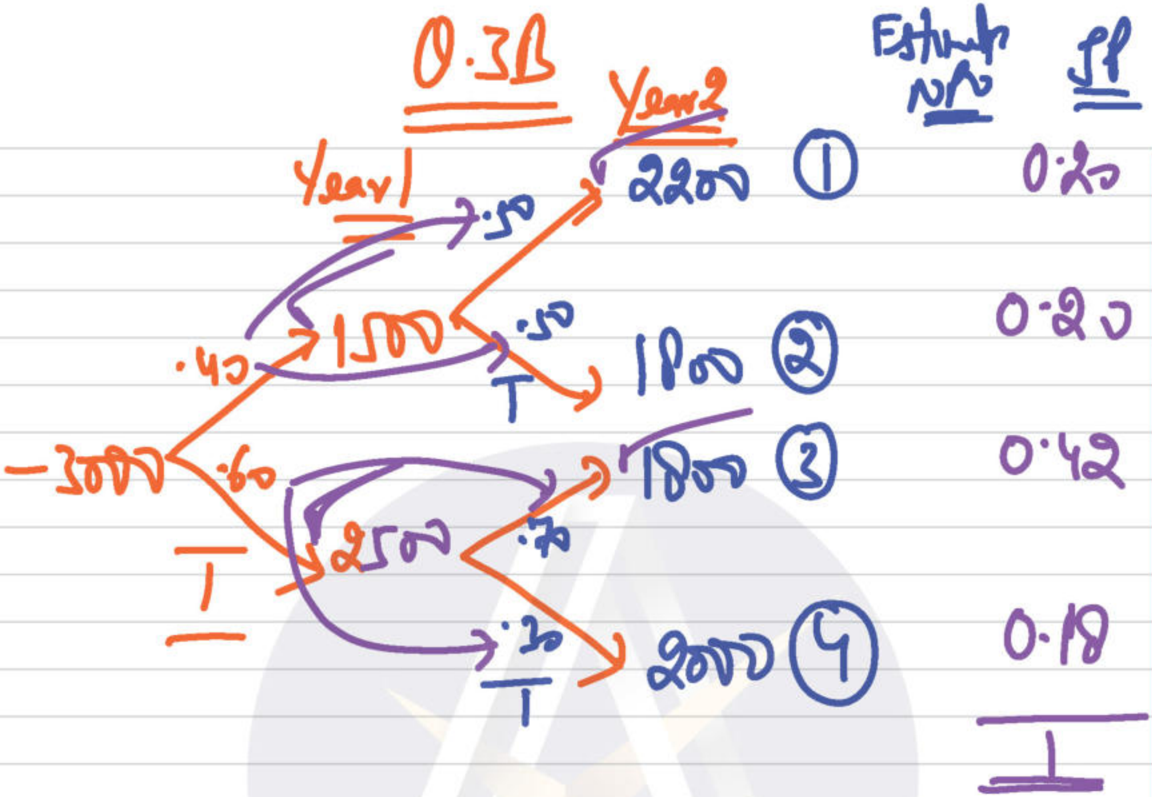
Extra Part:-

$$\boxed{\text{Expected Profit} = 12050 \times 6\% \Rightarrow 723}$$

(d) Cal. of Expected NPV:-

Estimate NPV (X)	JP.	Expected NPV
−7363	0.08	−7363 × 0.08
−4059	0.12	−4059 × .12
897	0.20	897 × .20
3790	0.24	3790 × .24
7920	0.30	7920 × .30
12050	0.06	12050 × .06
		$\bar{X} \Rightarrow 3111.88$

Since, Expected NPV is positive, project should be accepted.



W.No.1 Cal. of Estimate NPV

① $-3000 + \dfrac{1500}{(1+.12)^1} + \dfrac{2200}{(1+.12)^2} \Rightarrow (+)\ 92.90$

$.893 \qquad .797$

② $-3000 + \dfrac{1500}{(1+.12)^1} + \dfrac{1800}{(1+.12)^2} \Rightarrow (-)\ 225.90$

(3) $-3000 + \frac{2500}{(1.12)^1} + \frac{1800}{(1.12)^2} \Rightarrow (+)\,667.10$

(4) $-3000 + \frac{2500}{(1.12)^1} + \frac{2000}{(1.12)^2} \Rightarrow (+)\,826.50$

(b)

Estimate NPV (X)	JP	Exp. NPV	$(X-\bar{X})^2$ Prob.
92.90	.20	$92.90 \times .20$	$(92.90 - 402.35)^2 \times .2$
−225.90	.20	$-225.90 \times .20$	$(-225.90 - 402.35)^2 \cdot .2$
667.10	.42	$667.10 \times .42$	$(667.10 - 402.35)^2 \cdot .4$
826.50	.18	$826.50 \times .18$	$(826.50 - 402.35)^2 \times .$
	1	$\bar{X} = 402.35$	$\Sigma \Rightarrow 159913$

$$S.D = \sqrt{159913}$$

$$\Rightarrow 399.89 \text{ or } \underline{\underline{400}}$$

Mean NPV is ₹402.35. Project has a positive NPV, it should be accepted.

The S.D. of NPV is ₹400.

Q.3C V.V.Imp Type 2

$r = 1$

	Prob.	1-5 years	+ 5th year		Estimate NCF	JP
-4,00,000	.30	1,00,000	20,000	(1)	✓	.30
	.50	1,10,000	50,000	(2)	✓	.50
	.20	120,000	60,000	(3)	✓	.20
	1					1

W.No.1 Cal. of JP:-

$.30 \times 1 \times 1 \times 1 \times 1 = .30$

$.50 \times 1 \times 1 \times 1 \times 1 \Rightarrow .5$

$.20 \times 1 \times 1 \times 1 \times 1 \Rightarrow .20$

W.No.2 Cal. of Estimate NPV:-

3.791

(1) $-4{,}00{,}000 + 1{,}00{,}000 \times PVAF @ 10\%, 5 \text{ years}$
$+\ 20{,}000 \times PVF @ 10\%, 5^{th} \text{ year}$

0.621

$\Rightarrow$ (+) 8480

(2) $-4,00,000 + 1,10,000 \times PVAF @ 10\%, 5\,year$

$+$

$50,000 \times PVF @ 10\%, 5^{th}\,year$

$\Rightarrow$ (+) 48060

(3) $-4,00,000 + 1,20,000 \times PVAF @ 10\%, 5\,year$

$+$

$60,000 \times PVF @ 10\%, 5^{th}\,year$

$\Rightarrow$ (+) 92,180

Alt. 1

Expected NPV: –

Estimate NPV	JP	Exp. NPV
−8480	·30	−8480 × ·30
+48060	·50	+48060 × ·50
+92180	·20	+92180 × ·20
	1	$\bar{X} \Rightarrow 39922$

Alt. 2

Expected Cash Outflows:–

$\Rightarrow$ 4,00,000 × ·30 + 4,00,000 × ·50 + 4,00,000 × ·20

$\Rightarrow$ 4,00,000

Expected Cash Inflows:-

$\Rightarrow$ 1,00,000 × .30 + 1,10,000 × .50
+ 1,20,000 × .20 $\Rightarrow$ 109000

Expected SV:-

$\Rightarrow$ 20,000 × .30 + 50000 × .50
+ 60,000 × .20 $\Rightarrow$ 43000
[End of 5th year]

Expected NPV:-

− 4,00,000 + 109000 × PVAF @ 10%, 5 year
+
43000 × PVF @ 10%. 5th year

$\Rightarrow$ (+) 39922

(ii) Best Case NPV ⇒ +92180

Worst Case NPV ⇒ (–) 8480

(iii) (a) The Prob. of occurrence of worst case if cash flows are perfectly dependent over time.

r = 1

$0.30 \times 1 \times 1 \times 1 \times 1 \Rightarrow 0.30$

(b) If CF's are independent over time:–

$.30 \times .30 \times .30 \times .30 \times .30$

$\Rightarrow (.30)^5$ or 0.00243

(iv) (X) $(X-\bar{X})^2$ Prob.

$-8480 \quad (-8480 - 39922)^2 \times .30$

$+48060 \quad (48060 - 39922)^2 \times .50$

$+92180 \quad (92180 - 39922)^2 \times .20$

$\sigma^2 \Rightarrow 1282119316$

$S.D = \sqrt{1282119316}$

$\boxed{\sigma \Rightarrow 35807}$

$CV = \dfrac{S.D}{\text{Exp.NPV}} \Rightarrow \dfrac{35807}{39922}$

$\Rightarrow 0.8969$

(v) Revised DR

$= 10\% - 1\% \Rightarrow \underline{\underline{9\%}}$

Revised Expected NPV:-

$\Rightarrow -4,00,000 + 109800 \times \underset{3.890}{PVAF @ 9\%, 5 yrs}$

$+ 43000 \times \underset{.650}{PVF @ 9\%, 5^{th} yr}$

$\Rightarrow -4,00,000 + 451960$

$\Rightarrow (+) \underline{\underline{51960}}$

Decision:

Since, the project NPV is +ve,

Project should be accepted

Type 3:-

In this type of question, we can't make decision tree i.e. in each year CF's are different.

Note:- Cash flows are dependent or not dependent is not relevant for Cal. of Exp. NPV.

However, it is relevant for Cal. of S.D. → Hiller's
→ Hertz's

Q.3E V.V.Imp.

W.No.1 Cal. of Expected CF's & Deviations

(₹ lakh)

Year 1

CFAT (X)	Prob	Exp CF's	$(X-\bar{X})^2$ Prob.
14	.10	14 × .10	$(14-27)^2 \times .10$
18	.20	18 × .20	$(18-27)^2 \times .20$
25	.40	25 × .40	$(25-27)^2 \times .40$
40	.30	40 × .30	$(40-27)^2 \times .20$
	1	$\bar{X} \Rightarrow 27$	$\Sigma = 85.40$

$\sigma^2 \Rightarrow 85.40$

$\sigma = \sqrt{85.40} \Rightarrow 9.24$

Year 2

CFAT (Y)	Prob.	Exp. CF's	$(Y-\bar{Y})^2$ Prob
15	.10	15×.10	$(15-29.30)^2 \times .10$
20	.30	20×.30	$(20-29.30)^2 \times .30$
32	.40	32×.40	$(32-29.30)^2 \times .40$
45	.20	45×.20	$(45-29.30)^2 \times .20$
	1	$\bar{Y} \Rightarrow 29.30$	$\Sigma = 98.61$

$\sigma^2 \Rightarrow 98.61$

$\sigma = \sqrt{98.61} \Rightarrow 9.93$

Year 3

CFAT (Z)	Prob.	Exp. CF's	$(Z-\bar{Z})^2 \times$ Prob.

18	·20	$18 \times \cdot 20$	$(18-27\cdot90)^2 \times \cdot20$
25	·50	$25 \times \cdot 50$	$(25-27\cdot90)^2 \times \cdot50$
35	·20	$35 \times \cdot 20$	$(35-27\cdot90)^2 \times \cdot20$
48	·10	$48 \times \cdot 10$	$(48-27\cdot90)^2 \times \cdot10$
	1	$\Sigma \Rightarrow 27\cdot90$	$\Sigma = 74\cdot29$

$\sigma^2 \Rightarrow 74\cdot29$

$\sigma = \sqrt{74\cdot29} \Rightarrow 8\cdot62$

(i) Expected NPV:- (£ lakh)

0	1	2	3
−50	$\frac{27}{(1+\cdot06)^1}$	$\frac{29\cdot3}{(1+\cdot06)^2}$	$\frac{27\cdot9}{(1+\cdot06)^3}$

Year	Exp CF's	PVF@6%	PV
1	27	0.943	25.461
2	29.30	0.890	26.077
3	27.90	0.840	23.436
		PV of CF's	74.974
Less	Cash outflow		50.000
		Expected NPV	(+) 24.974

Type 3: Cal. of S.D.

Cash flows

Independent CF's

⇓

Hiller's Model

⇓

Year 1	Year 2	Year 3
$\frac{\text{Variance}_1}{(1+r)^{1\times 2}}$	$\frac{\text{Variance}_2}{(1+r)^{2\times 2}}$	$\frac{\text{Variance}_3}{(1+r)^{3\times 2}}$

↳ Double Discounting.

⇒ Variance as on today

$= S.D$ as on today $= \sqrt{\text{Variance}}$.

Dependent CF's

(Perfectly Correlated)

⇓

Hertz's Model

①	②	③
$\frac{S.D_1}{(1+r)^1}$	$\frac{SD_2}{(1+r)^2}$	$\frac{SD_3}{(1+r)}$

⇒ S.D as on today

(ii) Hillier's Model [CF's are Independent]

$$\frac{\text{Variance}_1}{(1+r)^{1\times 2}} \quad \frac{\text{Variance}_2}{(1+r)^{2\times 2}} \quad \frac{\text{Variance}_3}{(1+r)^{3\times 2}}$$

$\Rightarrow \sqrt{\text{Variance as on today.}}$

$\Rightarrow$ S.D. as on today.

$$\frac{85.40}{(1+.06)^{1\times 2}} \quad \frac{98.61}{(1+.06)^{2\times 2}} \quad \frac{74.29}{(1+.06)^{3\times 2}}$$

$\Rightarrow$ **206.49** $\Rightarrow$ Variance as on today.

$= \sqrt{206.49}$ = S.D as on today

$\Rightarrow \underline{\underline{14.37}}$ ✓

Extra Part:

If CF's are dependent (Perfectly Correlated)

$$\frac{S.D_1}{(1+r)^1} + \frac{SD_2}{(1+r)^2} + \frac{SD_3}{(1+r)^3}$$

$$\Rightarrow \frac{9.24}{(1+.06)^1} + \frac{9.93}{(1+.06)^2} + \frac{8.62}{(1+.06)^3}$$

$\Rightarrow$ 24.79

(iii) S.D. is a statistical measure of dispersion, it measures the risk under the capital Budgeting decision.

When we have two or more project having similar CF's or Expected NPV, by Cal. S.D. we can measure the extent of variation.

If two projects are identical in respect of CF's & NPV, we should consider the less risky project for execution.

Note: As per ICAI

If question is silent about CF's whether CF's are independent or dependent, we always assume Independent CF's & Use Hillier's **Model**. **(Cal. of S.D.)**

ACB

Concept:- Sensitivity Analysis:-

"What if" Analysis

(Adverse Effect) (Negative Impact)

Factor's Effecting the NPV:- (Adverse Effect)

(i)	Cash Inflows	⇓
(ii)	Cash Outflows	⇑
(iii)	Sale Price / Unit	⇓
(iv)	V/C / Unit	⇑
(v)	Fixed Cost	⇑
(vi)	Discount Rate	⇑

(vii) Life ⇓.

(viii) Sales Volume ⇓.

⇒ It is the tool in the hands of the firm to analyse the change in the project NPV for a given change in one of the Variable for which NPV is calculated provided other factor's are constant.

Drawback:-

It considers only single factor at a time, keeping other factor's constant.

It assumes factors are independent. So, you can change one factor keeping other factor's constant.

Practically this may not hold good as factors are interdependent like Sales Price & Sales Quantity.

In such case, Sensitivity Analysis fails.

Two Approaches to Calculate Sensitivity :-

1) MOS [Margin of Safety Approach]

2) Shock Approach.

1) **MOS:- Margin of Safety Approach:-**

⇒ Set NPV = 0 & Calculate the Break-even Values & Margin of Safety for each factor at one time, keeping other factors constant.

⇒ The Most critical factor is that factor for which Margin of Safety is least.

Q.5A

(i) Existing NPV:-

$\Rightarrow 40{,}000 \times PVAF @ 10\%, 4\text{ years} - 1{,}00{,}000$ (3.170)

$\Rightarrow 40{,}000 \times 3.170 - 1{,}00{,}000$

$\Rightarrow$ ₹ 26,800

(ii) (a) Sensitivity to Size of Investment:-

$\underbrace{40{,}000 \times PVAF @ 10\%, 4\text{ years}} - x = 0$

$x = 126800$

$\boxed{1{,}00{,}000 \longrightarrow 126800} \longrightarrow NPV = 0$

MOS = 126800 − 1,00,000 = 26800

$\% \Rightarrow \frac{\text{Change}}{\text{Base}} \times 100$

$\Rightarrow \frac{126800 - 1,00,000}{1,00,000} \times 100$

$\Rightarrow \underline{\underline{26.80\%}}$

ⓑ **Sensitivity to Cash Inflows :-**

$x \times \text{PVAF @ } 10\%, \text{4 years} - 1,00,000 = 0$ (3.170)

$x = \frac{1,00,000}{3.17} \Rightarrow \underline{\underline{₹ 31546 \text{ p.a.}}}$

$\text{MOS} = 40,000 - 31546$

$\Rightarrow \underline{\underline{8454}}$

$\%\ \text{Sensitivity} \Rightarrow \frac{40,000 - 31546}{40,000} \times 100$

$\Rightarrow$ 21.14%

(iii) Sensitivity to life:-

$40{,}000 \times PVAF @ 10\%, x \text{ years} - 1{,}00{,}000 = 0$

$\Downarrow$

Discounted PBP

Cal. DPBP:-

Year	CF's	PVF@10%	PV	Cumulative
1	40,000	·909	36360	36360
2	40,000	·826	33040	69400
3	40,000	·751	30040	99440
4	40,000	·683	27320	126760

$$3 \text{ years} + \frac{1,00,000 - 99440}{27320}$$

DPBP ⇒ 3.02 years → NPV = 0

% Sensitivity ⇒ $\frac{4 - 3.02}{4} \times 100$

⇒ 24.5%

(d) Sensitivity to Discount Rate:-

40,000 × PVAF @ x%, 4 years − 1,00,000 = 0

IRR → DR at which NPV = 0

NPV @ 20%:-

40,000 × PVAF @ 20%, 4 yrs − 1,00,000

2.589

$\Rightarrow (+)\ \underline{\underline{3560}}$

NPV @ 25%

$40{,}000 \times \text{PVAF @ } 25\%, 4\text{ yrs} \overset{2.362}{} - 1{,}00{,}000$

$\Rightarrow (-)\ \underline{\underline{5520}}$

$$IRR = LR + \frac{LR_{NPV}}{LR_{NPV} - HR_{NPV}} \times \text{Diff in rate}$$

$$\Rightarrow 20\% + \frac{3560}{3560 + 5520} \times 5$$

$\Rightarrow \underline{\underline{21.96\%}}$ p.a. $\longrightarrow$ $\underline{\underline{NPV = 0}}$

$$\%\ \text{Sensitivity} \Rightarrow \frac{21.96 - 10}{10} \times 100$$

$\Rightarrow \underline{\underline{119.60\%}}$

⇒ The most critical factor is cash inflows as MOS is least in this case.

& least critical factor is Discount Rate

Q.5D V.V.Imp.

(i) IRR = 16%

It is a DR at which NPV = 0

or PV of CI's = PV of CO's / Initial Invst

$\Downarrow$

DR = 16% = IRR

> ⇒ If we Discount CI's @ IRR i.e. 16%, this is equal to the PV of CO's / Initial Invst.

⇒ ₹ 57500 × PVAF @ 16%, 5 years = Initial Investment

⇒ ₹ 57500 × 3.274 ⇒ ₹ 188255

(ii) Net Present Value:-

Sensitivity of DR = 60%

$$DR \uparrow 60\% \longrightarrow NPV = 0 \longrightarrow IRR = 16\%$$

If DR = x

$$x + x \times 60\% = 16\% \text{ (IRR)}$$

$$\Rightarrow x(1 + .60) = 16\%$$

$$\Rightarrow \boxed{x \Rightarrow 10\%} \Rightarrow \text{Discount Rate}$$

NPV = PV of CI's − Initial Invt.

$$\Rightarrow 57500 \times PVAF\ @\ 10\%,\ 5\ years$$

(−)

1,88,255

$\Rightarrow 57500 \times 3.791 - 188255$

$\Rightarrow ₹\ 29728$

(iii) Annual Fixed Cost:-

Annual Fixed Cost Sensitivity = 7.8416%

If Annual fixed Cost = x

If Annual fixed Cost Increase by 7.8416%
Than NPV became Zero

It means:-

Increase in fixed Cost = 29728

$x \times 7.8416\% \times PVAF @ 10\%, 5 years = 29728$

$x \times 0.078416 \times 3.791 = 29728$

$$\boxed{x \Rightarrow 1,00,000 \text{ p.a.}}$$

Annual fixed cost = 1,00,000 p.a.

(iv) Estimated Annual Sales Units:-

$[SP - VC]$ No. of units $-$ Fixed Cost = Cash Inflows

$[200 - 60]\ x - 1,00,000 = 57,500$

$$\boxed{x \Rightarrow 1125 \text{ units}}$$

W.N:-

PV Ratio = 70%

V/C Ratio = 30% V/C pu = 60/unit

Sales Price pu $\Rightarrow \frac{60}{30\%} = ₹200/\text{unit}$

(V) Break-even Units:-

$$\Rightarrow \frac{FC}{\text{Contribution/unit}}$$

$$\Rightarrow \frac{1,00,000}{(200-60)} \Rightarrow \underline{\underline{715 \text{ units}}}$$

Method 2:

Shock Approach:-

Eg: 10% → Adverse Direction

CI's ⇒ ⇓ 10% → Revised NPV

CO's ⇒ ⇑ 10% → Revised NPV

DR = ⇑ 10% → Revised NPV

[Least NPV
or
[% fall in NPV]$_{max.}$] Most Critical factor

⇒ Shock each critical Risk factor in

The adverse direction like 5% / 10% / 15% & find the revised NPV. or % falling NPV.

$$\boxed{\%\text{ fall in NPV} = \frac{\text{Change}}{\text{Base}} \times 100}$$

Decision Criteria:-

Revised NPV ⇒ least

or

% fall in NPV = Maximum.

most critical factor

Q.5C [SM]

Cal. of Existing NPV:-

Year	CF	PVF@10%	PV
0	(1,20,000)	1	(1,20,000)
1-4	45000	3.169	142605
			22605

(i) If initial Outflow is varied adversely by 10%:-

Revised Initial Outflow:-

= 1,20,000 + 10%

⇒ ₹ 132000

Revised NPV :-

$\Rightarrow 45000 \times 3.169 - 132000$

$\Rightarrow$ ₹ 10605

% fall in NPV $\rightarrow \frac{22605 - 10605}{22605} \times 100$

$\Rightarrow$ 53.09%

(ii) If cash inflows are varied adversely by 10% :-

Revised Cash Inflows :-

45000 − 10% of 45000

$\Rightarrow$ ₹ 40500

Revised NPV ⇒ 40500 × 3.169 − 120000

⇒ 8345

% fall in NPV ⇒ $\frac{22605 - 8345}{22605} \times 100$

⇒ 63.08%

(iii) If cost of capital is adversely affected by 10% :-

Revised CoC ⇒ 10% + 10% of 10%

⇒ 11%

Revised NPV :-

45000 × PVAF @ 11%, 4 years

$(-)$
$1,20,000$

$\Rightarrow 45000 \times 3.103 - 1,20,000$

$\Rightarrow \underline{\underline{19635}}$ ✓

% fall in NPV $\Rightarrow \dfrac{22605 - 19635}{22605} \times 100$

$\Rightarrow \underline{\underline{13.14\%}}$

Decision:-

The most critical factor is Cash Inflows, it has maximum adverse effect on NPV by $\underline{\underline{63.08\%}}$

Q.5B [SM]

Using Shock Approach:- **Shock of 10%**
(In Adverse Direction)

1) **Cal. of Existing NPV:-**

Year	CF's		PVF @10%	PV
1	20,000 [60-40] = 4,00,000		.909	363600
2	30,000 [60-40] = 6,00,000		.826	495600
3	30,000 [60-40] = 6,00,000		.751	450600
				1309,800

Existing NPV ⇒ 1309800 — 10,00,000
= **309800**

(a) If Sales Price/Unit decrease by 10%:-

Revised Sales Price/Unit = 60 − 10%

⇒ 54/Unit

Revised NPV:-

Year	CFs		PVF@10%	PV
1	20,000 × 14	⇒ 2,80,000	.909	254520
2	30,000 × 14	⇒ 4,20,000	.826	346920
3	30,000 × 14	⇒ 4,20,000	.751	315420
				916860

NPV ⇒ 916860 − 10,00,000

⇒ (−) 83140

% fall in NPV ⇒ $\frac{309800 + 83140}{309800} \times 100$

⇒ 126.84% fall in NPV

(b) Unit Cost:

If unit cost increased by 10%

Revised unit Cost = 40 + 10% of 40

⇒ 44/unit

Cal. of Revised NPV:-

Year	CF's		PVF@10%	PV
1	20,000 × 16	⇒ 320,000	.909	290880
2	30,000 × 16	⇒ 480,000	.826	396480

3 $30{,}000 \times 16 \Rightarrow 480{,}000$ $\cdot 751$ 360480

$\overline{10{,}47{,}840}$

Revised NPV $\Rightarrow$ $10{,}47{,}840 - 10{,}00{,}000$

$\Rightarrow$ $\underline{47{,}840}$

% fall in NPV:-

$$\Rightarrow \frac{309800 - 47840}{309800} \times 100$$

$\Rightarrow$ $\underline{84.56\%}$

ⓒ **If Sales Volume decrease by 10%:-**

Revised Sales Volume:-

Year

1 20,000 − 10% ⇒ 18000

2 30,000 − 10% ⇒ 27000

3 30,000 − 10% ⇒ 27000

Revised NPVs −

Year	CF's		PVF@10%	PV
1	18000 × 20	⇒ 360,000	·909	327240
2	27000 × 20	⇒ 540,000	·826	446040
3	27000 × 20	⇒ 5,40,000	·751	405540
				11,78,820

Revised NPV = 1178820 − 10,00,000

= 178820

$$\% \text{ fall in NPV} = \frac{309800 - 178820}{309800} \times 100$$

$$\Rightarrow \underline{\underline{42.28\%}}$$

(d) If Initial Outlay increase by 10%

$$\text{Revised Initial Outlay} = 10,00,000 + 10\% = \underline{\underline{11,00,000}}$$

Revised NPV :-

$$1309800 - 11,00,000 = \underline{\underline{209800}}$$

$$\% \text{ fall in NPV} \Rightarrow \frac{309800 - 209800}{209800} \times 100$$

$\Rightarrow$ 32.28%

(e) Project life time:- (ICAI)

⇓

Discounted PBP

Year	CF^s	PVF@10%	PV	Cumulative
1	4,00,000	.909	363600	363600
2	6,00,000	.826	495600	859200
3	6,00,000	.751	450600	1309800

$$\Rightarrow 2 \text{ years} + \frac{1900000 - 859200}{450600}$$

$\Rightarrow$ 2.31 years or 2 year & 114 Days

Thus, if Project runs for 2 years 4 114 Days then AE would be achieved by representing fall of $\frac{3-2.31}{3} \times 100$

$= 23\%$

The most critical factor is Sales Price/Unit.

Q.5E (SM)

Cal. of Existing NPV :-

Year 1

		PVF@9%	PV
Running cost	4000	.917	(3668)
Savings	12000	.917	11004

Year 2

Running cost	5000	.842	(4210)
Savings	14000	.842	11788

Cal. of NPV :-

PV of Savings − PV of Running cost − Initial Investment

⇒ 22792 − 7878 − 10,000

$\Rightarrow$ 4914

(i) Sensitivity to Initial Investment:-

To make NPV = 0, Initial Invt. may go up by 4914 — NPV = 0

$$\% \text{ Sensitivity} \Rightarrow \frac{4914}{10{,}000} \times 100$$

$$\Rightarrow 49.14\%$$

(ii) Sensitivity to Recurring Cost:-

To make NPV = 0, the PV of Recurring cost i.e. R_1 & R_2 which is presently 7878 may go up by £4914

$x \Rightarrow 7878 + 4914 \Rightarrow \underline{\underline{12792}}$

% Sensitivity $\Rightarrow \dfrac{12792 - 7878}{7878} \times 100$

$\Rightarrow \underline{\underline{62.38\%}}$

(iii) Sensitivity to Savings:-

To make NPV = 0, PV of Savings must go down by ₹ 4914

$x \Rightarrow 22792 - 4914 \Rightarrow \underline{\underline{17878}}$

% Sensitivity $\Rightarrow \dfrac{4914}{22792} \times 100$

$\Rightarrow \underline{\underline{21.56\%}}$

Hence, Savings are the most critical factor to affect the acceptability of the project.

Scenario Analysis:-

Q.4

Cal. of NPV under Different Scenarios:-

(₹ 000)

Worst Case

Year	CF's	PVF@9%	PV
0	(1400)	1	(1400)
1	450	.917	412.65
2	400	.842	336.80
3	700	.772	540.40
			(–) 110.15

Most Likely:-

Year	CF's	PVF@9%	PV

0	(1400)	1	(1400)
1	550	.917	504.35
2	450	.842	378.90
3	800	.772	617.60
			(+) 100.85

Best Case:-

Year	CF's	PVF @ 9%	PV
0	(1400)	1	(1400)
1	650	.917	596.05
2	500	.842	421.00
3	900	.772	694.80
			(+) 311.85

(ii)

	1st	2nd	3rd
−1400	550	450	700

Revised NPV:-

Year	CF's	DF@9%	PV
0	(1400)	1	(1400)
1	550	.917	504.35
2	450	.842	378.90
3	700	.772	540.40
			(+) 23.65
			(₹ 000)